

Perturbative-nonperturbative interference in the static QCD interaction at small distances

Yu.A.Simonov

Institute of Theoretical and Experimental Physics

Abstract

Short distance static quark–antiquark interaction is studied systematically using the background perturbation theory with nonperturbative background described by field correlators. A universal linear term $\frac{6N_c\alpha_s\sigma r}{2\pi}$ is observed at small distance r due to the interference between perturbative and nonperturbative contributions. Possible modifications of this term due to additional subleading terms are discussed and implications for systematic corrections to OPE are formulated.

1 Introduction

It is more than 20 years ago that the power correction has been computed in OPE [1] laying ground for numerous later applications in QCD. Since then OPE is the basic formalism for study of short–distance phenomena, such as DIS, e^+e^- annihilation and, with some modifications, heavy quark systems.

Interaction of static charges at small distances has drawn a lot of attention recently [2-4]. The theoretical reason is that the appearance of linear terms in the static potential $V(r) = \text{const } r$, where r is the distance between charges, implies violation of OPE, since $\text{const} \sim (\text{mass})^2$ and this dimension is not available in terms of field operators. There are however some analytic [5,6] and numerical arguments [7,8] for the possible existence of such terms $O(m^2/Q^2)$ in asymptotic expansion at large Q .

Of special importance is the sign of the mass squared term. It was argued recently in [3] that the small distance region may produce tachyonic mass correction and this correction was studied selfconsistently in different QCD

processes. In particular the correct (positive) sign of linear potential at small distances comes from tachyonic gluon mass, while positive mass squared term produces negative slope of linear potential.

On a more phenomenological side the presence of linear term at small distances, $r < T_g$, where T_g is the gluonic correlation length [9,10], is required by at least two sets of data.

First, the detailed lattice data [11] do not support much weaker quadratic behaviour of $V(r) \sim \text{const } r^2$, following from OPE and field correlator method [9,10], and instead prefer the same linear form $V(r) = \sigma r$ at all distances (in addition to perturbative $-\frac{C_2\alpha_s}{r}$ term). Second, the small-distance linear term is necessary for the description of the fine structure splittings in heavy quarkonia, since the spin-orbit Thomas term $V_t = -\frac{1}{2m^2r} \frac{dV}{dr}$ is sensitive to the small r region and additional linear contribution at $r < T_g$ is needed to fit the experimental splittings [12]. Moreover the lattice calculations [13] display $1/r$ behaviour for the spin-orbit potential V_1' in all measured region up to $r = 0.1fm$.

In what follows we display the basic dynamics which produces tachyonic gluon mass and estimate its magnitude.

2 Background perturbative theory

In this letter we report the first application of the systematic background perturbation theory [14] to the problem in question. One starts with the decomposition of the full gluon vector potential A_μ into nonperturbative (NP) background B_μ and perturbative field a_μ ,

$$A_\mu = B_\mu + a_\mu, \quad (1)$$

and the 'tHooft identity for the partition function

$$Z = \int DA_\mu e^{-S(A)} = \frac{1}{N} \int DB_\mu \eta(B) \int Da_\mu e^{-S(B+a)} \quad (2)$$

where $\eta(B)$ is the weight for nonperturbative fields, defining the vacuum averages, e.g.

$$\langle F_{\mu\nu}^B(x) \Phi^B(x, y) F_{\lambda\sigma}^B(y) \rangle_B = \frac{\hat{1}}{N_c} (\delta_{\mu\lambda} \delta_{\nu\sigma} - \delta_{\mu\sigma} \delta_{\nu\lambda}) D(x - y) + \Delta_1 \quad (3)$$

where $F_{\mu\nu}^B, \Phi^B$ are field strength and parallel transporter made of B_μ only; Δ_1 is the full derivative term [9] not contributing to string tension σ , which is

$$\sigma = \frac{1}{2N_c} \int d^2x D(x) + O(\langle FFFF \rangle) \quad (4)$$

The background perturbation theory is an expansion of the last integral in (2) in powers of ga_μ and averaging over B_μ with the weight $\eta(B_\mu)$, as shown in (3). Referring the reader to [14] for explicit formalism and renormalization, we concentrate below on the static interquark interaction at small r . To this end we consider the Wilson loop of size $r \times T$, where T is large, $T \rightarrow \infty$, and define

$$\langle W \rangle_{B,a} = \langle \text{Pexp}ig \int_C (B_\mu + a_\mu) dz_\mu \rangle_{B,a} \equiv \exp\{-V(r)T\} \quad (5)$$

Expanding (5) in powers of ga_μ , one obtains

$$\langle W \rangle = W_0 + W_2 + \dots; \quad V = V_0(r) + V_2(r) + V_4(r) + \dots, \quad (6)$$

where $V_n(r)$ corresponds to $(ga_\mu)^n$ and can be expressed through D, Δ_1 and higher correlators [10,14] and its behaviour at small r is [9]

$$V_0(r) = C_0 r^2 + C'_0 r^4 + \dots, \quad r \lesssim T_g, \quad V_0(r) = \sigma r, \quad r \gg T_g \quad (7)$$

where coefficients are integrals of field correlators over Euclidean time,

$$C_0 = O(\langle FF \rangle) = \int_0^\infty D(\nu) d\nu + O(\Delta_1), \quad C'_0 = O(\langle F^4 \rangle).$$

It is this small r behaviour (7) which causes phenomenological problems mentioned above.

Coming now to $V_2(r)$, describing one exchange of perturbative gluon in the background, one finds from the quadratic in a_μ term in $S(B+a)$ in the background Feynman gauge the gluon Green's function

$$G_{\mu\nu} = -(D_\lambda^2 \delta_{\mu\nu} + 2igF_{\mu\nu}^B)^{-1}, \quad D_\lambda^{ca} = \partial_\lambda \delta_{ca} + gf^{cba} B_\lambda^b \quad (8)$$

The term W_2 can be written through $G_{\mu\nu}$ as

$$W_2 = g^2 \int_0^T dx_4 \int_0^T dy_4 \langle \text{Pexp}(ig \int_C B_\mu dz_\mu) G_{44}(x, y) \rangle_B \quad (9)$$

where for G_{44} one can use the Feynman–Schwinger representation (FSR) [10]. The simplest form of FSR obtains when one can neglect or expand in powers of gluon spin interaction (paramagnetic term $2igF_{\mu\nu}^B$ in (8)).

Doing this expansion, $G_{\mu\nu}$ can be written as

$$G = -D^{-2} + D^{-2}2igF^B D^{-2} - D^{-2}2igF^B D^{-2}2igF^B D^{-2} \quad (10)$$

the first term on the r.h.s. of (10) corresponds to the spinless gluon exchange in the background B_μ , which can be written using FSR [10] as

$$G_{44}(x, y) = \int_0^\infty ds (Dz)_{xy} e^{-K} \text{Pexp}ig \int_{C(z)} B_\mu dz_\mu, \quad K = \frac{1}{4} \int_0^s \dot{z}^2 d\tau \quad (11)$$

Here B_μ is in the adjoint representation of $SU(N_c)$. At large N_c one can use the 'tHooft rule to replace the gluon adjoint trajectory $C(z)$ by the double fundamental trajectory which forms together with the original rectangular contour C in (9) two closed Wilson loops (see [14] for details and discussion).

The average over B_μ in (9) then reduces (again in the large N_c limit) to the product of two averaged Wilson loops, namely

$$W_2 = \int_0^\infty ds (Dz)_{xy} e^{-K} dx_4 dy_4 < W(C_1) > < W(C_2) >, \quad K = \frac{1}{4} \int_0^s \dot{z}^2 d\tau, \quad (12)$$

where C_1 and C_2 are two contours obtained from the rectangular Wilson loop when two points on it, x and y , are connected by a double line of gluon trajectory. It is convenient to choose the surfaces S_i inside C_1 and C_2 as consisting of two adjacent pieces $S_i(C_i) = S_i(\text{plane}) + S(\Delta)$, one lying on the plane of original Wilson loop and another piece Δ , perpendicular to the plane and bounded by the trajectory. Using now the nonabelian Stokes theorem [9] and cluster expansion for the average $< W(C_i) >$, one obtains that bilocal correlator of fields with points on the two pieces vanishes since $< E_i E_k > \sim \delta_{ik}$, and $< E_i B_k > = 0(\Delta_1)$ and vanishes by symmetry arguments. Trilocal and higher correlators for dimensional resons bring with them higher powers of distance r and can be neglected.

Hence one has

$$< W(C_1) > < W(C_2) > = < W_\Delta >^2 W_0 \quad (13)$$

where

$$W_\Delta = \exp\left(-\frac{1}{2N_c} \int_\Delta D(x-y) d\sigma_{\mu\nu}(x) d\sigma_{\mu\nu}(y)\right) \quad (14)$$

Two different regimes are possible for (14). In the small distance region, $r \lesssim T_g$, the sizes of the surface Δ are of the order of r and one can replace $D(x - y) \rightarrow D(0)$ in (14), and for dimensional reasons the only possible contribution is

$$W_\Delta \cong 1 + O(D(0)r^4) \quad (15)$$

Thus one obtains a correction $O(r^4)$ to the perturbative potential $1/r$, and hence no linear term.

In the large distance region, $r \gg T_g$, one obtains the area law for W_Δ ,

$$W_\Delta \cong \exp(-\sigma S_\Delta) \quad (16)$$

Insertion of (16) in (12) yields a massive propagator of a spinless hybrid with mass $m = m_\Delta$ at large r , which corresponds to the first excitation of the open string with fixed ends.

Summarizing one can rewrite W_2 as

$$W_2 = W_0 \int dx_4 \int dy_4 < G(x, y) > \quad (17)$$

where $G(x, y) >$ is the Green's function of the spinless hybrid. One can satisfy the properties (15), (16) representing $< G >$ as the propagator of a particle with variable mass $m_0(p)$,

$$< G(p) > = \frac{1}{p^2 + m_0^2(p)} \quad (18)$$

where $m_0^2(p) \lesssim O(\frac{1}{p^2})$, $p \rightarrow \infty$, and $m_0^2(p \rightarrow 0) = m_\Delta^2$.

Thus at small distances (large p) $< G(p) >$ describes the usual massless gluon exchange, whereas at large distances it describes the propagation of the spinless hybrid.

Hence at small $r \lesssim T_g$ the background field B_μ in D^{-2} is not operative and one can replace D^{-2} by the free gluon propagator ∂^{-2} . The contribution of the second term to W_2 , $dx_\mu D^{-2} F_{\mu\nu}^B D^{-2} dy_\nu$, obtains when x and y are on adjacent sides of the Wilson loop and therefore does not affect $V(r)$. In what follows we concentrate on the third term in (10), $W_2^{(3)}$,

$$\begin{aligned} W_2^{(3)} &= 4g^2 \int_0^T dx_4 \int_0^T dy_4 < G(x, u) > d^4 u < g^2 F_{4i}^B(u) F_{i4}^B(v) > \\ &\times < G(u, v) > d^4 v < G(v, y) > \end{aligned} \quad (19)$$

One can rewrite (19) as

$$W_2^{(3)} = 2g^2 \int_0^T dx_4 \int_0^T dy_4 \int \frac{d^4 k e^{ik(x-y)}}{(k^2 + m_0^2(k))^2 (2\pi)^4} \lambda^2(k), \quad (20)$$

where we have defined, having in mind (4)

$$\mu^2(k) = 3 \int \frac{D(z) e^{-ikz} d^4 z}{4\pi^2 z^2}; \mu^2(0) = \frac{3\sigma N_c}{2\pi}, \quad (21)$$

Doing integrals over $dx_4 dy_4$, one gets

$$W_2^{(3)} = \frac{T}{\pi^2} \int \frac{d^3 k e^{i\vec{k}\vec{r}} \alpha_s(k) \lambda^2(k)}{(\vec{k}^2 + m_0^2(k))^2} = -\Delta V_2(r) T \quad (22)$$

To estimate the integral (22) one can take $\alpha_s(k) \lambda^2(k)$ out of integral at some effective point k_0 and calculate the rest in a simple way, assuming m_0 to be constant, and expanding result at small r .

In this way one obtains

$$\Delta V_2(r) = \frac{\alpha_s(k_0) \lambda^2(k_0)}{m_0 r} \frac{\partial}{\partial m_0} e^{-m_0 r} = \alpha_s(k_0) \mu^2(k_0) \left(-\frac{1}{m_0} + r + O(r^2) \right) \quad (23)$$

Analysis of (21) tells that $\lambda^2(k)$ is a rather weak function of the argument, and to get an idea of the magnitude of $\Delta V_2(r)$, one can approximate $\lambda^2(k_0) \cong \lambda^2(0) = \frac{3\sigma N_c}{2\pi}$ yielding for $\Delta V_2(r)$,

$$\Delta V_2(r) \sim \frac{3N_c \alpha_s \sigma r}{2\pi}, \quad r \lesssim T_g. \quad (24)$$

Note, that had we renormalized α_s in (23), (24) in the standard way we would meet the IR divergence of the running $\alpha_s(k)$, since the corresponding momentum k_0 is in the IR regime for the constant term $(-\frac{1}{m_0})$ in (23). However, the NP background formalism predicts IR modification of α_s (see [14,15] for details and discussion), the so-called freezing α_s behaviour, which from heavy quarkonia fitting was found in [16] to yield maximal α_s at small k ,

$$\alpha_s(max) = 0.5 \quad (25)$$

For the linear term in (23) the situation is different¹ and the effective value of k_0 is of the order of $1/r$. Hence it is more appropriate to present (23) in the form

$$\Delta V_2(r) = \alpha_s(1/r) \mu^2(1/r) r \quad (26)$$

¹The author is grateful to V.I.Zakharov for the discussion of this point

One may wonder whether behaviour (24) holds also at large r , thus increasing linear potential $V_0(r)$. However the form (24) was obtained at small $r \lesssim T_g$, while at large r next terms of expansion in (10) are contributing and the whole series (10) should be summed up explicitly.

One can perform the summation, replacing first for simplicity $D \rightarrow \partial$ in (8), (small r or relatively large q are considered) and assuming that only bilocal correlators of $F_{\mu\nu}$ are nonzero. One obtains

$$G = -(\partial^2 + 2igF)^{-1} \rightarrow G(k) = \frac{1}{k^2 - \mu^2(k)},$$

where $\mu^2(k) > 0$. Thus $G(k)$ acquires a pole at real value $\mu^2(k)$ in Euclidean space-time, signalling appearance of a tachyon. From physical point of view this result is a consequence of paramagnetic attractive interaction of gluon spin with NP background, yielding negative correction to the gluon self-energy. A similar term occurs for a quark due to its spin interaction with background [17].

Recently a negative (tachyonic) mass shift was observed due to the gluon interaction with the stochastic background in [18].

It is meaningful, that the same paramagnetic term $F_{\mu\nu}^B$ in $G_{\mu\nu}$ yields negative contribution to the charge renormalization (asymptotic freedom) [19]. In fact negative paramagnetic effective action $S_{eff}^{(para)}$ [19] gives rise to the negative (tachyonic) mass since both are connected, $-\mu^2\delta_{\mu\nu} \sim \frac{\delta^2 S}{\delta a_\mu \delta a_\nu}$. Therefore one may expect that the phenomenon of tachyonic gluon mass is pertinent to nonabelian theories.

The existence of the tachyon reveals paramagnetic instability of the object (gluon or quark), if stabilizing mass is not created by some additional mechanism. In our case this mechanism is the creation of hybrid mass due to the same confining correlator $D(x)$ when D^2 is used and not ∂^2 in (8). As was explained above, the hybrid mass is created at larger distances, $r \gg T_g$, so that for illustrative purposes (referring the hybrid mass to the gluon in question), one may write the total gluon propagator $G(k)$ as

$$G(k) = \frac{1}{k^2 + m^2(k)},$$

where $m^2(k) = m_0^2(k) - \mu^2(k)$, and $m_0^2(k)$ is dominating at small k (large distances) while $\mu^2(k)$ dominates at large k (small distances). A simple example is provided by $m^2(k) = \mu^2 \frac{\mu^2 - k^2}{\mu^2 + k^2}$, in which case one can calculate

gluon exchange potential $V(r) = - \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\vec{r}} G(k)$ explicitly to yield at small r :

$$V(r) \sim -\frac{1}{r} + \frac{\mu^2}{2}r, \quad \Delta V \sim \frac{\mu^2}{2}r$$

in agreement with the form (24). At large distances one should calculate the exact gluon Green's function (8) in the Wilson loop $W^{(2)}$, which describes to the propagation of the hybrid state with two static quarks $Q\bar{Q}$ at the ends of the string.

Such a state was considered both analytically [20] and on the lattice [21] yielding excitation energy (which corresponds to the mass $m(0)$) around 1 GeV.

Hence (24) is only a small distance approximation of the hybrid exchange potential, where the dominant paramagnetic contribution is kept in the effective gluon mass.

3 Other possible corrections to $V(r)$

In doing perturbative expansion in (2) one encounters other terms in $S(B+a)$ which potentially yield interference contributions of perturbative a_μ and NP B_μ fields. Of special importance is the term L_1 ,

$$L_1 = \int a_\nu D_\mu(B) F_{\mu\nu}^B d^4x \quad (27)$$

Correction due to (27) in the gluon propagator was studied in Appendix 1 of [14] and can be written in the form (20) where $\mu^2(k^2) \rightarrow J_1$ is now expressed as

$$J_1(k^2) = \int d^4z e^{-ikz} \langle DF^B(z) DF^B(0) \rangle \quad (28)$$

Now the integrand in (28) can be written as a sum of two terms [22],

$$\langle D_\rho F_{\rho\nu}^B(z) D_\lambda F_{\lambda\mu}^B(u) \rangle = \partial_\rho \partial_\lambda \langle F^B(z) F^B(u) \rangle + 0(\langle FFF \rangle) \quad (29)$$

The first term on the r.h.s. of (29) yields $\delta J_1 \sim k^2$ and hence only a NP correction to the Coulomb term, while the second term, $\langle FFF \rangle$, would give, using dimensional arguments, correction $\Delta V \sim \langle FFF \rangle r^5$, negligible at small r .

Another type of correction occurs from the interplay of multiple color Coulomb exchanges and one NP contribution, considered in [23]. This effect can be accounted for by the replacement

$$D(x, t) \rightarrow D(x, t) \exp\left(-\frac{N_c \alpha_s t}{2x}\right) \quad (30)$$

Here t is the Euclidean time and the exponent in (30) accounts for the difference of potential in singlet and octet channels. Insertion of (30) in (7) yields an additional suppression of the r^2 dependence. This result coincides with the correction obtained in [24] in a different way.

Finally we consider in this section the correction due to the freezing behaviour of the coupling α_s [14,15],

$$\alpha_s^f(r) = \frac{4\pi}{b_0 \ln \frac{1+r^2 m_B^2}{\Lambda^2 r^2}} \cong \alpha_s^{(0)}(r) - \frac{\alpha_s^{(0)} m_B^2 r^2}{\ln \frac{1}{\Lambda^2 r^2}} \quad (31)$$

As was noticed by F.J.Yndurain [2], expansion of the freezing Coulomb potential yields a linear term

$$\Delta V_c(r) = + \frac{C_2 \alpha_s^{(0)} m_B^2 r}{\ln \frac{1}{\Lambda^2 r^2}} + \dots \quad (32)$$

Here m_B is the double hybrid mass, ($m_B = 1.1 GeV$ from the fits to experiment [16]), and the term (32) is always much smaller than the Coulomb term because of condition $m_B^2 r^2 \ll 1$. One may consider ΔV_c in (32) as coming from the additional $1/p^2$ in $\alpha_s(p)$, as was suggested in [8], but here there is the log term in the denominator, reminding that the pole is coming from the expansion of the freezing $\alpha_s(p)$.

One should note that there is no double counting in adding (24) and (32), since (24) is obtained from the one-gluon exchange (OGE) process, while (32) is due to the one-loop corrections to OGE. However the region of validity of (32) is always smaller than (24).

4 Discussion and conclusion

The analysis done heretofore concerns static interquark potential and reveals that even at small distances NP background ensures some contributions which come from relatively small intermediate distances, $l \lesssim T_g$, and

encoded in the mass squared term

$$\mu^2 = \frac{3\sigma N_c}{2\pi}, \mu \approx 0.5 GeV \quad (33)$$

which generates the potential (24).

Applying the same NP background formalism to other processes of interest, one would get similar corrections of the order of $\frac{\mu^2}{p^2}$, as e.g. in OPE for e^+e^- annihilation.

One example of this kind is the calculation of the field correlator $\langle F_{\mu\nu}(x)\Phi(x,y)F_{\lambda\sigma}(y) \rangle$. The leading contribution can be written using the gluon propagator G (8),

$$\langle F\Phi F \rangle = \partial_\mu \partial_\lambda G_{\nu\sigma} + perm + G_{\mu\lambda} G_{\nu\sigma} + perm \quad (34)$$

where we have suppressed Φ and perm. denotes terms obtained by the permutation of indices with the proper change of sign. Insertion of expansion (10) into (34) yields in addition to the standard perturbative term $O((x-y)^{-4})$ a contribution proportional to (33),

$$\langle F(x)\Phi F(0) \rangle \sim \frac{C_1}{x^4} + \frac{C_2\mu^2}{x^2} + \dots \quad (35)$$

where both C_1 and C_2 are positive computable numbers.

A recent lattice study of a similar quantity [7] as a function of UV cut-off Λ reveals the possible presence of the $O(\Lambda^2 \sim x^{-2})$ term.

It is clear that appearance of μ^2 , which is an integral of nonlocal entity $\langle F(x)F(0) \rangle$ over a NP scale, $x \sim T_g$, violates the original OPE of Wilson [25], proved in the pure perturbation theory, and the extended OPE of Shifman, Vainshtein and Zakharov [26], where NP contributions enter as matrix elements of local operators.

This extended form of OPE can be considered as a physically motivated assumption, and an explicit treatment done here within the background perturbation theory (BPT) reveals that some extra terms should be added to OPE, the first of which, μ^2/p^2 , was discussed in [3].

One might ask at this point, how rigorous and selfconsistent is BPT, with nonperturbative background given by correlators. One should stress here, that BPT is a consistent and systematic method, but not a complete one, since no recipe was suggested above for calculation of NP correlators,

and the NP configurations are introduced by hand just as it is done in QCD sum rules [26].

However recently the situation has changed. In [27] equations have been derived in the limit of large N_c for vacuum correlators, from which correlators can be computed one by one explicitly. In this way the exponential form of the lowest correlator, $\langle F(x)\Phi F(0) \rangle$ was defined analytically [27] in agreement with lattice studies, and analytic connection between T_g and σ was found, yielding T_g in a good agreement with lattice data [28].

The main conclusion from these studies is that NP configurations appear as a selfconsistent solution of nonlinear equations which violates spontaneously scale symmetry pertinent to these equations and their perturbative solutions.

From this point of view the NP background exploited here can be identified with scale violating NP solutions in [27], and the BPT method is made complete.

On the phenomenological side the account of the correction (24) and (32) in the total potential

$$V(r) = V_0(r) - \frac{C_2\alpha_s^f(r)}{r} + \Delta V_2(r)$$

may provide for $V(r)$ a simple "linear plus Coulomb" picture for all distances which is in better agreement both with experiment [12,17] and with lattice data [11, 13].

To conclude: in this study perturbative – nonperturbative interference was shown to provide additional OPE terms, absent in the usual local OPE form. In addition there are purely NP contributions [14] which are also outside of the standard lore, and will be discussed elsewhere.

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References

- [1] V.A.Novikov, L.B.Okun, M.A.Shifman, M.B.Voloshin, M.I.Vysotsky and V.I.Zakharov, Phys. Rep. **C41** (1978) 1.

- [2] F.J.Yndurain, hep-ph/9708448, Nucl. Phys. (proc. Suppl.) **64** (1998) 433;
R.Akhoury and V.I.Zakharov, Phys. Lett. **B438** (1998) 165.
- [3] K.G.Chetyrkin, S.Narison and V.I.Zakharov, hep-ph/9811275.
- [4] F.V.Gubarev, M.I.Polikarpov and V.I.Zakharov, hep-ph/9812030, ITEP-TH-73/98;
V.I.Zakharov, hep-ph/9811294.
- [5] G.Grunberg, hep-ph/9705290, hep-ph/9705460, hep-ph/9711481.
- [6] V.I.Zakharov, Nucl. Phys. **B385** (1992) 385;
A.I.Vainshtein, V.I.Zakharov, Phys. Rev. Lett. **73** (1994) 1207; Phys. Rev. **D54** (1996) 4039.
- [7] G.Burgio, F.Di Renzo, G.Marchesini, E.Onofri, Phys. Lett. **B422** (1998) 219.
- [8] G.Burgio, F.Renzo, C.Parrinello, C.Pittori, hep-ph/9808258, hep-ph/9809450.
- [9] H.G.Dosch and Yu.A.Simonov, Phys. Lett. **B205** (1988) 339;
for a review see Yu.A.Simonov, Physics Uspekhi **39** (1996) 313, hep-ph/9709344.
- [10] Yu.A.Simonov, Nucl. Phys. **B324** (1989) 67;
Yu.A.Simonov, Nucl. Phys. **B307** (1988) 512 and refs. therein.
- [11] S.P.Booth et al. Phys. Lett. **B294**, (1992) 385;
G.Bali, K.Schilling, A.Wachter, hep-lat/9506017.
- [12] A.M.Badalian, V.P.Yurov, Yad. Fiz. **56** (1993) 239;
A.M.Badalian and Yu.A.Simonov, Yad. Fiz. **59** (1996) 2247.
- [13] K.D.Born et al. Phys. Lett. **B329** (1994) 332;
G.S.Bali, K.Schilling, A.Wachter, Phys. Rev. **D56** (1997) 2566.
- [14] Yu.A.Simonov, Yad. Fiz. **58** (1995) 113; JETP Letters, 57 (1993) 525
Yu.A.Simonov, in: Lecture Notes in Physics, Springer, v.479, 1996.

- [15] A.M.Badalian, Phys. At. Nuclei, **60** (1997) 1003, hep-lat/9704004;
A.M.Badalian, Yu.A.Simonov, Phys. At. Nuclei **60** (1997) 630.
- [16] A.M.Badalian, V.L.Morgunov, hep-ph/9901430.
- [17] H.G.Dosch, Yu.A.Simonov, preprint HD-THEP-92-23; Yad.Fiz. **57**
(1994) 152
- [18] S.J.Huber, A.Laser, M.Reuter and M.G.Schmidt, Nucl. Phys. **B** (to be
published)
- [19] A.M.Polyakov, Gauge Fields and Strings, Hardwood, Chur, 1987, chap-
ter 2.
- [20] M.Luescher, Nucl. Phys. **B180** (1981) 317;
Yu.A.Simonov, in Proc. of Hadron'93 Conference (Como, 21-25 June
1993); ed. T.Bressani, A.Feliciello, G.Preparata and P.G.Ratcliffe;
Yu.S.Kalashnikova, Yu.B.Yufryakov, Phys. Lett. **B359** (1995) 175;
Yad. Fiz. **60** (1997) 374.
- [21] S.Perantonis, C.Michael, Nucl. Phys. **B347** (1990) 854;
T.Manke et al. CP-PACS Collaboration, hep-lat/9812017.
- [22] V.I.Shevchenko, Yu.A.Simonov, Phys. At. Nucl. **60** (1997) 1201.
- [23] Yu.A.Simonov, S.Titard and F.J.Yndurain, Phys. Lett. **B354** (1995)
435.
- [24] I.I.Balitsky, Nucl. Phys. **B254** (1985) 166.
- [25] K.Wilson, Phys. Rev. **179** (1969) 1499.
- [26] M.Shifman, A.Vainshtein, V.Zakharov, Nucl. Phys. **B147** (1978) 385.
- [27] Yu.A.Simonov, Phys. At. Nucl. **61** (1998) 855; hep-ph/9712250.
- [28] A.Di Giacomo, H.Panagopoulos, Phys. Lett. **B285** (1992) 133;
A.Di Giacomo, E.Meggiolaro, H.Panagopoulos, Nucl. Phys. **B483**
(1997) 371;
M.D'Elia, A.Di Giacomo, E.Meggiolaro, Phys. Lett. **B408** (1997) 315;
A.Di Giacomo, M.D'Elia, H.Panagopoulos, E.Meggiolaro, hep-
lat/9808056.